

## Inhibition of abnormal grain growth in BaTiO<sub>3</sub> by addition of Al<sub>2</sub>O<sub>3</sub>

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### Abstract

The effect of additions of up to 1 mol% Al<sub>2</sub>O<sub>3</sub> on abnormal grain growth in BaTiO<sub>3</sub> samples sintered at 1200 and 1250 °C has been studied. Samples with and without additions of 0.4 mol% TiO<sub>2</sub> were prepared. For the samples without added TiO<sub>2</sub>, addition of 0.1 mol% Al<sub>2</sub>O<sub>3</sub> increases the number density of abnormal grains, with further additions reducing the number density. The initial increase in number density is caused by Al<sub>2</sub>O<sub>3</sub> forming a solid solution with BaTiO<sub>3</sub> and releasing TiO<sub>2</sub> to the grain boundaries. This excess TiO<sub>2</sub> then reacts with BaTiO<sub>3</sub> to form Ba<sub>6</sub>Ti<sub>17</sub>O<sub>40</sub>, which promotes {1 1 1} twin formation and abnormal grain growth. Further additions of Al<sub>2</sub>O<sub>3</sub> react with BaTiO<sub>3</sub>, Ba<sub>6</sub>Ti<sub>17</sub>O<sub>40</sub> and excess TiO<sub>2</sub> to form Ba<sub>4</sub>Al<sub>2</sub>Ti<sub>10</sub>O<sub>27</sub> and BaAl<sub>2</sub>O<sub>4</sub> second phases, neither of which are growth sites for abnormal grains. For the samples with added TiO<sub>2</sub>, addition of Al<sub>2</sub>O<sub>3</sub> decreases the number of abnormal grains due to the Al<sub>2</sub>O<sub>3</sub> reacting with the excess TiO<sub>2</sub> and BaTiO<sub>3</sub> to form Ba<sub>4</sub>Al<sub>2</sub>Ti<sub>10</sub>O<sub>27</sub> and BaAl<sub>2</sub>O<sub>4</sub> instead of Ba<sub>6</sub>Ti<sub>17</sub>O<sub>40</sub>.

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### 1. Introduction

BaTiO<sub>3</sub> is an important dielectric ceramic with many uses such as capacitors and positive temperature coefficient of resistance (PTCR) materials. In these applications, control of the microstructure is necessary to optimize properties. The microstructure is dependant on processing factors such as sintering temperature and the presence of dopants or impurities. The BaTiO<sub>3</sub>–TiO<sub>2</sub> system has a BaTiO<sub>3</sub>–Ba<sub>6</sub>Ti<sub>17</sub>O<sub>40</sub> eutectic at 1332 °C.<sup>1</sup> When BaTiO<sub>3</sub> is sintered at temperatures above this eutectic point primary abnormal grain growth (PAGG) takes place.<sup>2,3</sup> Abnormally large grains nucleate and grow, consuming the fine matrix grains. This abnormal grain growth (AGG) is very rapid and the original fine-grained matrix can be completely replaced by coarse grains after 1 h of sintering at 1355 °C.<sup>4</sup> After PAGG is

completed the microstructure appears to have a log normal size distribution. If the sintering temperature is between 1360 and 1370 °C, then secondary abnormal grain growth (SAGG) can follow PAGG: some of the coarse primary grains begin to grow rapidly and consume the other primary grains.<sup>4</sup> This behaviour is linked to the presence of (1 1 1) twins.<sup>4,5</sup>

At temperatures below the eutectic, abnormal grain growth takes place.<sup>3,6–8</sup> Some of the matrix grains grow to a large size, forming a bimodal microstructure of fine matrix grains and large abnormal grains. Although AGG can be extensive in samples sintered for long periods below the eutectic temperature, complete consumption of the original matrix grains does not take place.

Both PAGG and AGG are affected by dopants. The eutectic point mentioned above can be lowered to ~1250 °C by the addition of Al<sub>2</sub>O<sub>3</sub>–SiO<sub>2</sub>–TiO<sub>2</sub>,<sup>9</sup> leading to PAGG at lower sintering temperatures.<sup>2</sup> When BaTiO<sub>3</sub> is doped with donor concentrations exceeding 0.3–0.5 at.% and sintered above the eutectic temperature, PAGG does not take place.

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This is known as the grain growth anomaly<sup>10</sup> or grain growth inhibition threshold.<sup>11</sup> Both A- and B-site donors effectively suppress PAGG.<sup>12</sup>

Below the eutectic, different additives can promote or suppress AGG, each according to its own mechanism. AGG is promoted by excess TiO<sub>2</sub>.<sup>13</sup> TiO<sub>2</sub> reacts with BaTiO<sub>3</sub> to form Ba<sub>6</sub>Ti<sub>17</sub>O<sub>40</sub>, which acts as a nucleation site for {1 1 1} twins.<sup>13</sup> These twins provide growth sites for abnormal grains by the twin-plane re-entrant edge mechanism.<sup>5,14,15</sup> For {1 1 1} twins to nucleate, the Ba<sub>6</sub>Ti<sub>17</sub>O<sub>40</sub> phase must be faceted (atomically smooth).<sup>13,16</sup> Excess TiO<sub>2</sub> is often added to BaTiO<sub>3</sub> as a sintering aid. If the excess TiO<sub>2</sub> is decreased to zero, then AGG is eliminated and normal grain growth occurs.<sup>17</sup> La<sub>2</sub>O<sub>3</sub> has been found to suppress AGG below the eutectic temperature by the formation of La<sub>2</sub>Ti<sub>2</sub>O<sub>7</sub> precipitates which scavenge excess TiO<sub>2</sub>.<sup>11</sup> ZnO suppresses AGG by segregation at the grain boundaries which decreases the grain boundary mobility.<sup>18</sup>

Although the effect of donor dopants on the grain growth of BaTiO<sub>3</sub> has been extensively studied, the effect of acceptor dopants is less well known. Acceptor dopants are often present as impurities and are sometimes added as sintering aids<sup>19</sup> and to modify the PTCR properties of BaTiO<sub>3</sub>.<sup>19,20</sup> Na<sub>2</sub>O inhibits AGG in Ti-excess BaTiO<sub>3</sub> sintered at 1215 °C by the formation of Na<sub>4</sub>TiO<sub>4</sub> precipitates, which scavenge excess TiO<sub>2</sub>.<sup>21</sup> Čeh and Kolar<sup>22</sup> found that CaO caused a decrease in primary abnormal grain size in samples sintered at 1450 °C and attributed this to the formation of Ba<sub>3</sub>Ca<sub>2</sub>Ti<sub>2</sub>O<sub>9</sub> at the grain boundaries. However, as Ca<sup>2+</sup> substituted for Ti<sup>4+</sup> in their work, the reduction in grain size may have been due to the excess Ti rather than the second phase.<sup>3</sup>

Al<sub>2</sub>O<sub>3</sub> has been added to BaTiO<sub>3</sub> as an acceptor dopant<sup>23,24</sup> and as a sintering aid in conjunction with SiO<sub>2</sub> and TiO<sub>2</sub>.<sup>25,26</sup> Addition of Al<sub>2</sub>O<sub>3</sub>–SiO<sub>2</sub>–TiO<sub>2</sub> has been found to reduce the primary abnormal grain size, leading to a finer microstructure,<sup>2</sup> and to suppress AGG.<sup>25</sup> Although the effect of Al<sub>2</sub>O<sub>3</sub>–SiO<sub>2</sub>–TiO<sub>2</sub> addition on microstructure development is well known, the effect of Al<sub>2</sub>O<sub>3</sub> by itself has not been studied extensively. Cheng<sup>26</sup> found that addition of 0.3 mol% Al<sub>2</sub>O<sub>3</sub> to SiO<sub>2</sub>-doped BaTiO<sub>3</sub> reduced the average grain size of samples sintered at 1350 °C and improved uniformity. On the other hand, Al-Allak et al.<sup>24</sup> found that addition of 0.55 mol% Al<sub>2</sub>O<sub>3</sub> to BaTiO<sub>3</sub> (with small amounts of SiO<sub>2</sub> and TiO<sub>2</sub> added as sintering aids) caused an increase in average grain size at sintering temperatures between 1260 and 1400 °C. Addition of Al<sub>2</sub>O<sub>3</sub> also caused the grain size distribution to widen, although AGG was absent in the samples. In both these works, SiO<sub>2</sub> and TiO<sub>2</sub> were added in addition to Al<sub>2</sub>O<sub>3</sub>.

Adding combinations of Al<sub>2</sub>O<sub>3</sub>, SiO<sub>2</sub> and TiO<sub>2</sub> makes it difficult to determine the effect that each separate additive has on abnormal grain growth behaviour. In the present investigation, we have therefore, studied the effect of Al<sub>2</sub>O<sub>3</sub> addition alone on abnormal grain growth in BaTiO<sub>3</sub>. The effect of Al<sub>2</sub>O<sub>3</sub> additions of up to 1 mol% on the AGG behaviour of BaTiO<sub>3</sub> sintered at temperatures of 1200 and 1250 °C is

described. TiO<sub>2</sub> is often added to BaTiO<sub>3</sub> as a sintering aid and so the effect of adding both Al<sub>2</sub>O<sub>3</sub> and TiO<sub>2</sub> has also been studied. The AGG behaviour of the Al<sub>2</sub>O<sub>3</sub>-doped samples can be explained by considering the effect of second phases.

## 2. Experimental

BaTiO<sub>3</sub> powders were prepared from Fuji Titanium HPBT-1 BaTiO<sub>3</sub>, Aldrich TiO<sub>2</sub> and Sumitomo AKP-50 Al<sub>2</sub>O<sub>3</sub>. Manufacturers' analyses of the powders are listed in Table 1. The Ba/Ti ratio of the BaTiO<sub>3</sub> powder is 0.997, i.e. there is a slight Ti excess. Two series of powders were prepared. The first series (series A) was doped with Al<sub>2</sub>O<sub>3</sub> only. Powders were prepared with additions of 0, 0.1, 0.2, 0.5 and 1.0 mol% Al<sub>2</sub>O<sub>3</sub>. The second series (series T) was doped with 0.4 mol% TiO<sub>2</sub> in addition to Al<sub>2</sub>O<sub>3</sub>. Powders were prepared with additions of 0, 0.1, 0.5 and 1.0 mol% Al<sub>2</sub>O<sub>3</sub>. The powders were ball milled in ethanol in polypropylene jars using ZrO<sub>2</sub> milling media. After milling, the slurries were dried, crushed and passed through a 150 μm sieve. Samples 9 mm in diameter and 4 mm thick were prepared by cold isostatic pressing at 200 MPa. Samples were placed on BaTiO<sub>3</sub> spacers in an alumina crucible and sintered in an alumina tube furnace. Samples were sintered at 1200 and 1250 °C for 10 or 100 h. Samples were pushed into or pulled out of the furnace over a period of 1 h. The heating and cooling rate was ~20 °C min<sup>-1</sup>.

Samples for optical microscopy and SEM were sectioned, polished to a 0.25 μm finish and etched in a solution of 1 vol.% HF–4 vol.% HCl–95 vol.% distilled H<sub>2</sub>O for ~5 s. Abnormal grain number density was measured from optical micrographs (Model DMLM, Leica Microsystems, Wetzlar, Germany). For each sample, the average number density and standard deviation was measured from 6 to 10 micrographs depending on the sample. Samples for SEM were gold coated and viewed in a Model XL 30 S FEG SEM (Philips, Eindhoven, Netherlands). The matrix grain size was measured using an image analyzing program (Matrox Inspector

Table 1  
Manufacturers' analyses of powders

Powder	Powder size (μm)	Purity (wt.%)	Impurities
BaTiO <sub>3</sub>	0.65	99.8	Ba/Ti mol ratio: 0.997 0.001 wt.% Fe <sub>2</sub> O <sub>3</sub> 0.002 wt.% Na <sub>2</sub> O <0.1 wt.% SrO <0.05 wt.% Al <sub>2</sub> O <sub>3</sub> <0.05 wt.% SiO <sub>2</sub>
TiO <sub>2</sub>	0.3	>99.9	
Al <sub>2</sub> O <sub>3</sub>	0.1–0.3	>99.99	<20 ppm Fe <sub>2</sub> O <sub>3</sub> <10 ppm Na <sub>2</sub> O <25 ppm Si <10 ppm Cu <10 ppm Mg

2.1, Matrox Electronic Systems Ltd., Dorval, Canada) and divided by 0.76 to determine the 3D grain size.<sup>27</sup> For wavelength dispersive spectroscopy (WDS), samples were polished and etched as above, carbon coated, and viewed in a Model 515 SEM (Philips, Eindhoven, Netherlands). Samples were prepared for transmission electron microscopy (TEM) by the standard method of sectioning, grinding, dimpling and ion beam milling. Samples were viewed in a JEM-3010 (JEOL, Tokyo, Japan) TEM at 300 kV.

Samples for optical and scanning electron microscopy (SEM) were heat-treated in flowing H<sub>2</sub> at 900 °C. This treatment stabilises the cubic phase at room temperature, removes the domain structure and makes the {1 1 1} twins more visible.<sup>28</sup> Samples for WDS and TEM did not receive this H<sub>2</sub> heat treatment.

### 3. Results

Fig. 1 shows optical micrographs of series A samples that have been sintered at 1250 °C for 100 h. Fig. 1(a) shows a sample with 0 mol% Al<sub>2</sub>O<sub>3</sub>. Abnormal grain growth has occurred throughout the sample, leading to a bimodal microstructure of fine matrix grains and large abnormal grains up to 200 μm in diameter. The abnormal grains are polyhedral with faceted boundaries. {1 1 1} single and double twins

(marked with arrows) are present in some of the abnormal grains but do not appear to affect the growth direction of the grains. Abnormal grains tend to congregate in clusters, although isolated abnormal grains are present. These clusters often have a ‘rosette’ appearance, with small abnormal grains in the centre of the cluster surrounded by larger abnormal grains.

Addition of 0.1 mol% Al<sub>2</sub>O<sub>3</sub> causes an increase in the number of abnormal grains (Fig. 1(b)). The abnormal grains are reduced in size, the maximum size being ~100 μm, but their shape remains unchanged. Likewise, twinning is present in some of the grains and abnormal grains still cluster together. Addition of 0.5 mol% Al<sub>2</sub>O<sub>3</sub> causes a large decrease in the number of abnormal grains (Fig. 1(c)). The maximum grain size of the abnormal grains increases to 200 μm and their average size increases. In the sample with 1.0 mol% Al<sub>2</sub>O<sub>3</sub> AGG is almost completely eliminated (Fig. 1(d)), the abnormal grains in the micrograph being the only ones visible in the plane of polish of this sample.

Fig. 2 shows optical micrographs of series T samples. Fig. 2(a) shows the sample with 0 mol% Al<sub>2</sub>O<sub>3</sub>. AGG has occurred to a much greater degree in this sample than in the equivalent series A sample with no added TiO<sub>2</sub> (Fig. 1(a)). The original fine-grained matrix has been almost completely consumed by abnormal grains, with matrix grains remaining in the gaps between abnormal grains. Both polyhedral

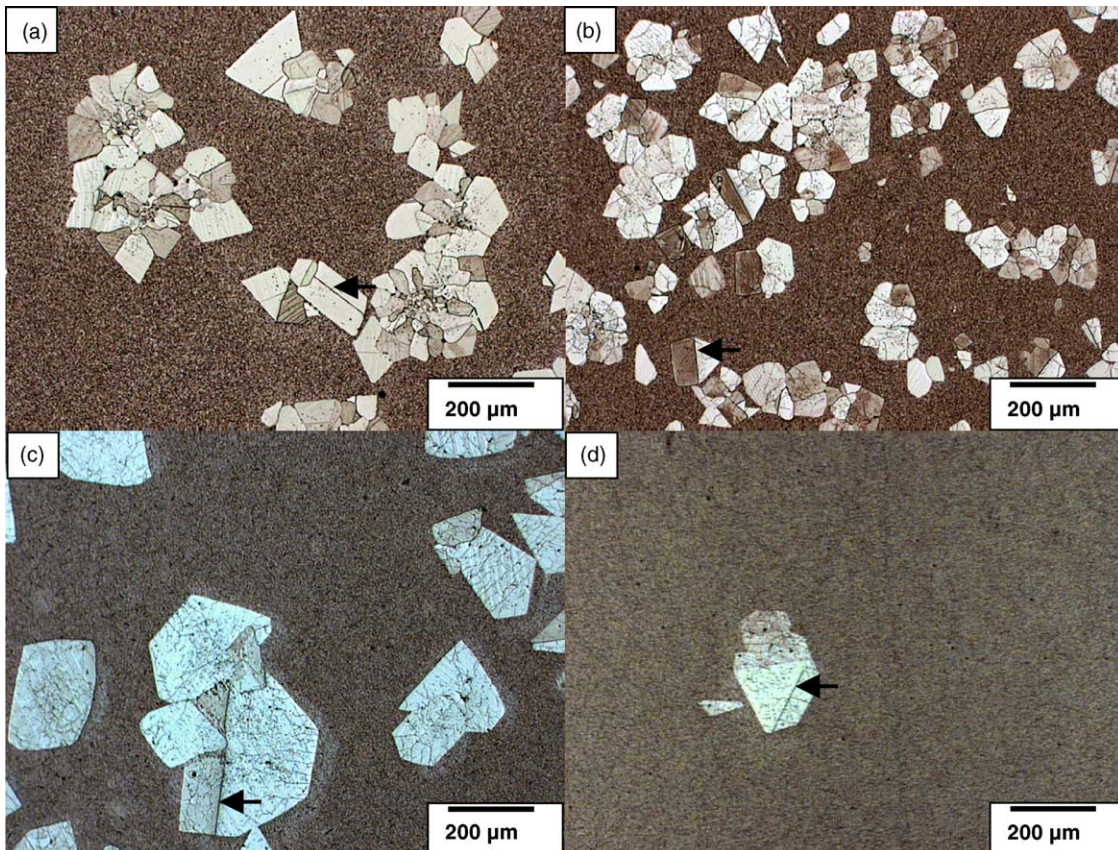


Fig. 1. BaTiO<sub>3</sub> samples doped with: (a) 0; (b) 0.1; (c) 0.5 and (d) 1.0 mol% Al<sub>2</sub>O<sub>3</sub> sintered at 1250 °C for 100 h.

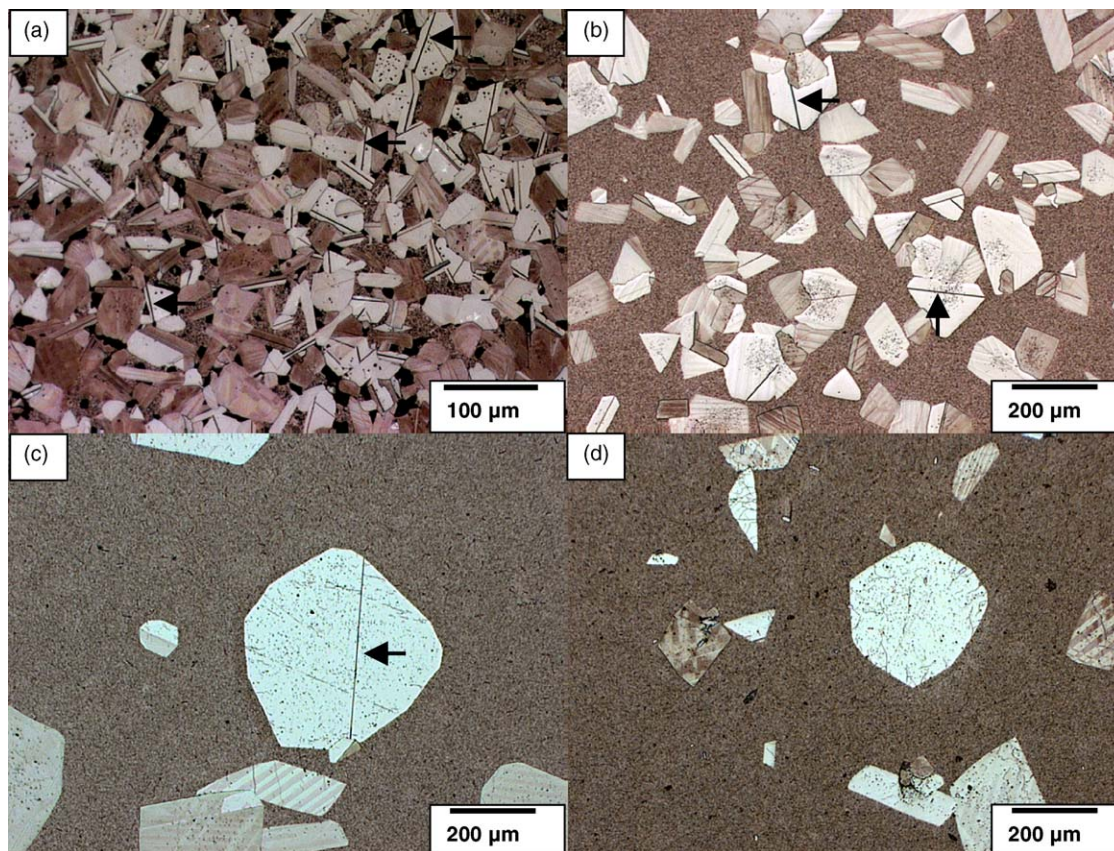


Fig. 2. BaTiO<sub>3</sub> 0.4 mol% TiO<sub>2</sub> samples doped with: (a) 0; (b) 0.1; (c) 0.5 and (d) 1.0 mol% Al<sub>2</sub>O<sub>3</sub> sintered at 1250 °C for 100 h.

and elongated abnormal grains are present. Single and double  $\{111\}$  twins are present in many of the abnormal grains (marked with arrows). In the elongated abnormal grains, the  $\{111\}$  twin runs parallel to the long face of the grain. In some of the polyhedral grains, two twins intersecting each other at an angle are visible.

Unlike the series A samples, addition of Al<sub>2</sub>O<sub>3</sub> immediately causes a sharp decrease in the number of abnormal grains in the series T samples (Fig. 2(b)). The size of the abnormal grains increases and the shape of the abnormal grains changes from both elongated and polyhedral to largely polyhedral. Further additions of Al<sub>2</sub>O<sub>3</sub> cause the number of abnormal grains to decrease further, with the number levelling out at 0.5 mol% Al<sub>2</sub>O<sub>3</sub>.

The samples of both series sintered at 1250 °C for 10 h follow the same patterns as above, although the extent of AGG is reduced. The samples of both series sintered at 1200 °C for 10 h show very little or no abnormal grain growth. The samples of both series sintered at 1200 °C for 100 h show appreciable abnormal grain growth. Abnormal grains in these samples are much smaller than abnormal grains in the corresponding samples sintered at 1250 °C. Abnormal grains in the samples sintered at 1200 °C are faceted and elongated. All the abnormal grains have single or double  $\{111\}$  twins running parallel to the long face of the grain. Some abnormal grains are polyhedral, with two  $\{111\}$  twins

intersecting each other at an angle. The series A samples sintered at 1200 °C for 100 h display similar behaviour to the samples sintered at 1250 °C: addition of 0.1 mol% Al<sub>2</sub>O<sub>3</sub> causes an increase in the number of abnormal grains, with further additions causing a decrease. The series T samples sintered at 1200 °C for 100 h also display similar behaviour to the samples sintered at 1250 °C: addition of Al<sub>2</sub>O<sub>3</sub> causes a decrease in the number of abnormal grains. However, the shape of the abnormal grains in the series T samples sintered at 1200 °C does not change from elongated to polyhedral with addition of Al<sub>2</sub>O<sub>3</sub>. Representative micrographs of samples sintered at 1200 °C are shown in Fig. 3.

The effect of Al<sub>2</sub>O<sub>3</sub> on the number density of abnormal grains in samples sintered for 100 h is shown in Fig. 4. For the series A samples, addition of 0.1 mol% Al<sub>2</sub>O<sub>3</sub> causes a large increase in the number density of abnormal grains. Further addition of Al<sub>2</sub>O<sub>3</sub> causes a rapid drop in the number density of abnormal grains. For the series T samples, addition of Al<sub>2</sub>O<sub>3</sub> causes a sharp decrease in the number density of abnormal grains, with the number density levelling out at 0.5 mol% Al<sub>2</sub>O<sub>3</sub>. Al<sub>2</sub>O<sub>3</sub> doping has the same effect on the samples regardless of sintering temperature. In the series A samples the sample doped with 0.1 mol% Al<sub>2</sub>O<sub>3</sub> shows a higher abnormal grain number density when sintered at 1200 °C than when sintered at 1250 °C. In the series T samples, the sample without Al<sub>2</sub>O<sub>3</sub> doping shows a higher abnormal grain number

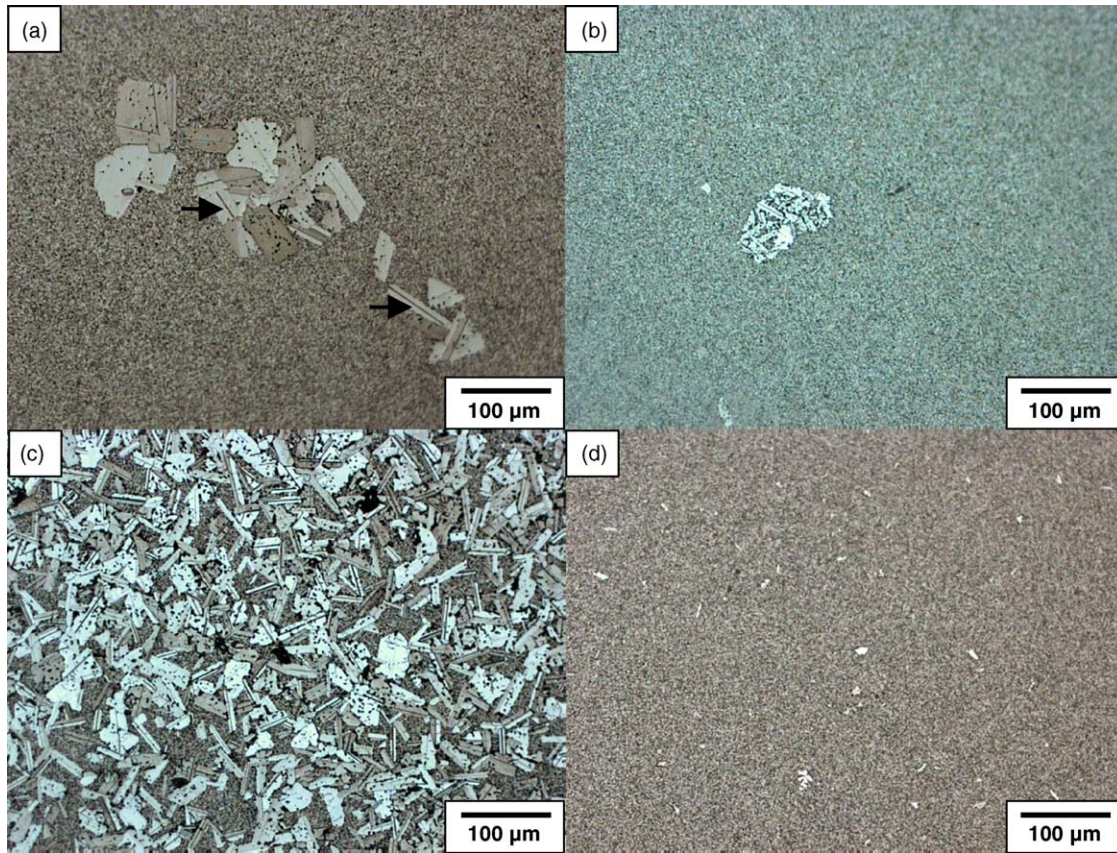


Fig. 3. BaTiO<sub>3</sub> samples doped with: (a) 0 mol% Al<sub>2</sub>O<sub>3</sub>; (b) 0.5 mol% Al<sub>2</sub>O<sub>3</sub>; (c) 0.4 mol% TiO<sub>2</sub>; (d) 0.4 mol% TiO<sub>2</sub> 0.5 mol% Al<sub>2</sub>O<sub>3</sub> sintered at 1200 °C for 100 h.

density when sintered at 1200 °C than when sintered at 1250 °C.

Fig. 5 shows TEM micrographs of matrix grains from series A samples. Micro-faceting (marked by arrows) is visible in all of the samples, irrespective of Al<sub>2</sub>O<sub>3</sub> content. Addition

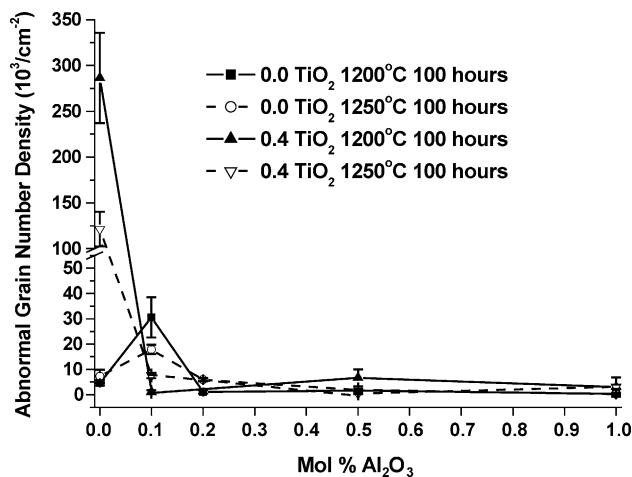


Fig. 4. Abnormal grain number density of BaTiO<sub>3</sub>-*x* mol% Al<sub>2</sub>O<sub>3</sub> and BaTiO<sub>3</sub>-0.4 mol% TiO<sub>2</sub>-*x* mol% Al<sub>2</sub>O<sub>3</sub> samples sintered at 1200 and 1250 °C for 100 h.

of Al<sub>2</sub>O<sub>3</sub> has not caused a roughening transition in BaTiO<sub>3</sub>. Also visible in Fig. 5(c) is an amorphous pocket at a triple junction (marked with a dotted circle).

Another amorphous pocket from a series A sample doped with 0.1 mol% Al<sub>2</sub>O<sub>3</sub> is shown in Fig. 6(a). Fig. 6(b) shows an amorphous pocket at a multi-grain junction in an A series sample doped with 1 mol% Al<sub>2</sub>O<sub>3</sub>. The selected area diffraction pattern (SADP) of the amorphous pocket is shown in the inset. Rings showing the presence of an amorphous phase are clearly seen. Diffraction spots from the surrounding grains are also visible. The amorphous phase indicates that a liquid phase was present during sintering. The presence of a liquid phase at a sintering temperature of 1250 °C is not expected from the compatibility phase diagram<sup>29</sup> but the presence of SiO<sub>2</sub> impurity in the BaTiO<sub>3</sub> powder has lowered the eutectic temperature to ~1250 °C.<sup>9</sup>

In both these samples the amorphous phase does not appear to have wetted the grain boundaries, as typically shown in the insert in Fig. 6(a). It is possible that the liquid phase wetted the grain boundaries during sintering and then dewetted during cooling of the samples. Choi et al.<sup>30</sup> found similar liquid pockets at triple junctions between matrix grains in TiO<sub>2</sub>-excess BaTiO<sub>3</sub> sintered at 1350 °C. They also found that if grain growth could be suppressed (in their case by pre-treatment of the samples in H<sub>2</sub> at 1250 °C), this liquid did

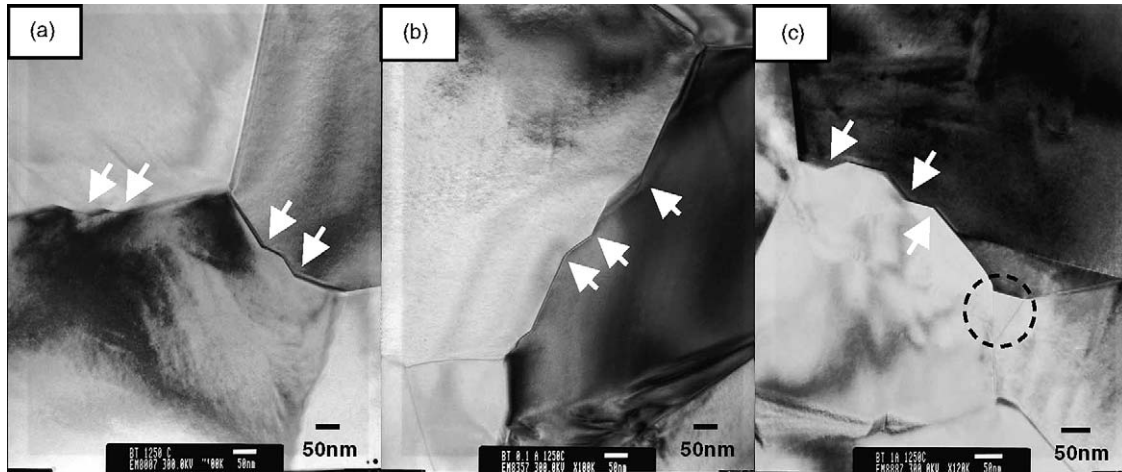


Fig. 5. TEM micrographs of: (a) 0; (b) 0.1 and (c) 1.0 mol%  $\text{Al}_2\text{O}_3$ -doped samples sintered at  $1250^\circ\text{C}$  for 100 h.

not penetrate the grain boundaries even after prolonged sintering at  $1350^\circ\text{C}$ . Moreover, they found that if abnormal grain growth and grain boundary wetting did take place, the liquid phase remained at the grain boundary and did not dewet upon cooling. In the present work, the liquid phase is also found at the triple junctions of matrix grains. These grains have undergone very limited grain growth and so it is likely that grain boundary wetting has not taken place. Plus, if a liquid phase had wetted the grain boundaries during sintering, it would be expected to remain at the grain boundaries during cooling, as shown in the work of Choi et al.<sup>30</sup>

Fig. 7 shows SEM micrographs of the samples. Fig. 7(a) shows a series T sample without  $\text{Al}_2\text{O}_3$  doping that was

sintered at  $1200^\circ\text{C}$  for 100 h. Matrix grains, elongated abnormal grains and a second phase (marked with an arrow) are present. WDS of this second phase identifies it as  $\text{Ba}_6\text{Ti}_{17}\text{O}_{40}$ . Fig. 7(b) shows a series A sample doped with 0.5 mol%  $\text{Al}_2\text{O}_3$  that was sintered at  $1200^\circ\text{C}$  for 100 h. Elongated abnormal grains containing  $\{111\}$  twins are present. Precipitates of a second phase (marked by arrows) are present both in abnormal grains and in the matrix. Precipitates appear in the series A samples when  $>0.1$  mol%  $\text{Al}_2\text{O}_3$  is added. WDS analysis of this second phase gives it a Ba/Al/Ti cation ratio close to 3/1/6. However, a compound with this cation ratio does not appear on the compatibility diagram of the  $\text{BaO}-\text{Al}_2\text{O}_3-\text{TiO}_2$  system, the closest phase being

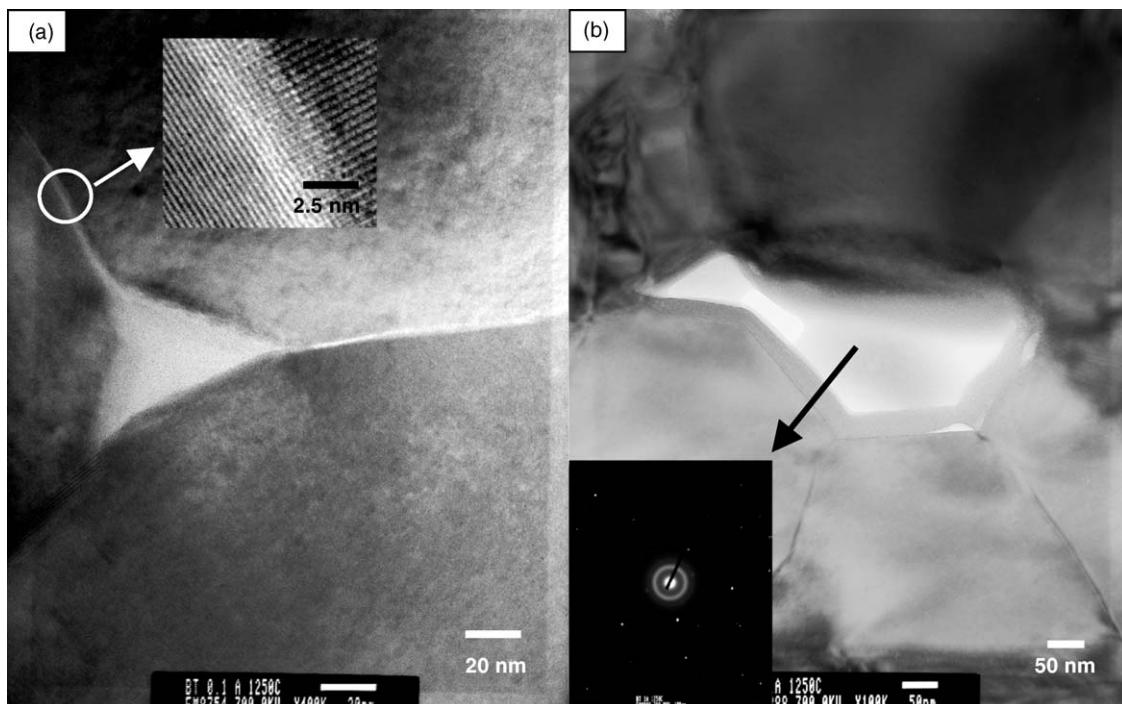


Fig. 6. TEM micrographs of amorphous phases in: (a) 0.1 and (b) 1.0  $\text{Al}_2\text{O}_3$ -doped samples sintered at  $1250^\circ\text{C}$  for 100 h.

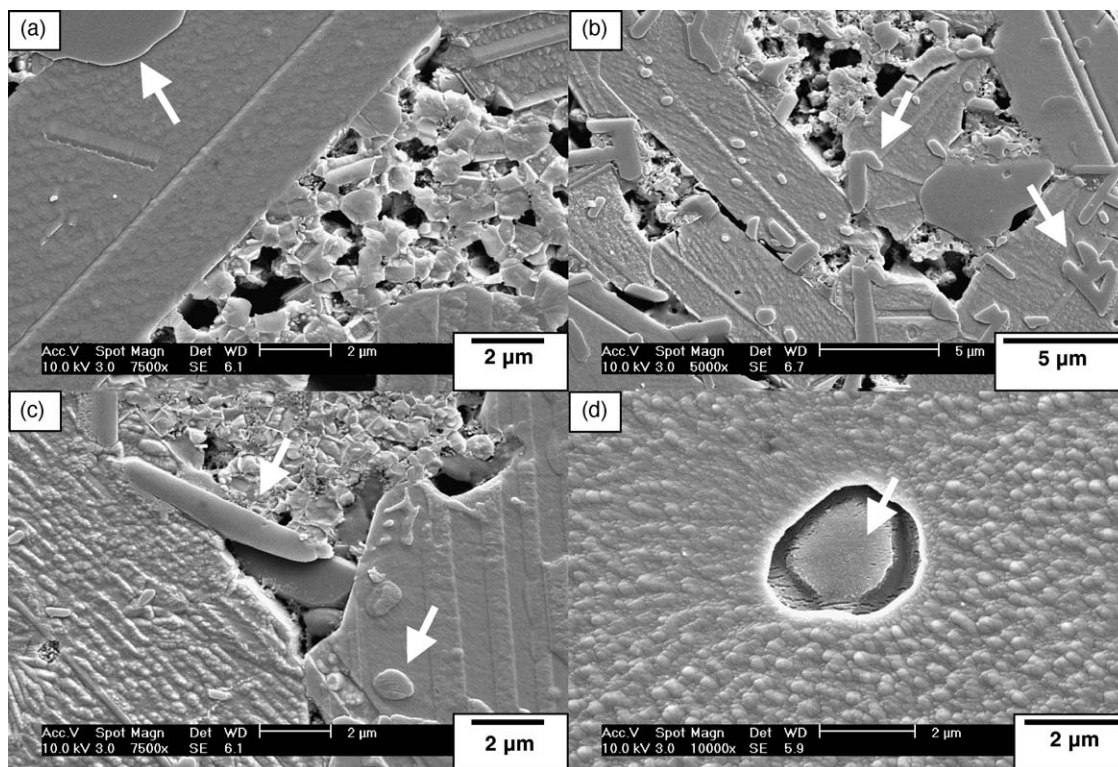


Fig. 7. SEM micrographs of: (a) 0.4 TiO<sub>2</sub>-doped sample sintered at 1200 °C for 100 h; (b) 0.5 Al<sub>2</sub>O<sub>3</sub>-doped sample sintered at 1200 °C for 100 h; (c) 0.4 TiO<sub>2</sub>-0.1 Al<sub>2</sub>O<sub>3</sub>-doped sample sintered at 1200 °C for 100 h; (d) 0.4 TiO<sub>2</sub>-0.5 Al<sub>2</sub>O<sub>3</sub>-doped sample sintered at 1250 °C for 10 h.

Ba<sub>4</sub>Al<sub>2</sub>Ti<sub>10</sub>O<sub>27</sub>.<sup>29</sup> The compositions sintered in this work lie on the border of the BaTiO<sub>3</sub>-Ba<sub>6</sub>Ti<sub>17</sub>O<sub>40</sub>-Ba<sub>4</sub>Al<sub>2</sub>Ti<sub>10</sub>O<sub>27</sub> and BaTiO<sub>3</sub>-Ba<sub>4</sub>Al<sub>2</sub>Ti<sub>10</sub>O<sub>27</sub>-BaAl<sub>2</sub>O<sub>4</sub> compatibility triangles so this second phase is likely to be Ba<sub>4</sub>Al<sub>2</sub>Ti<sub>10</sub>O<sub>27</sub>. In the series T samples, precipitates of the same composition appear in the samples as soon as Al<sub>2</sub>O<sub>3</sub> is added (Fig. 7(c)). Precipitates with the composition BaAl<sub>2</sub>O<sub>4</sub> also appear in the series A and series T samples doped with >0.1 mol% Al<sub>2</sub>O<sub>3</sub> (Fig. 7(d)). These BaAl<sub>2</sub>O<sub>4</sub> precipitates appear both in the abnormal grains and in the matrix. These Ba<sub>4</sub>Al<sub>2</sub>Ti<sub>10</sub>O<sub>27</sub> and BaAl<sub>2</sub>O<sub>4</sub> precipitates appear in samples sintered at both 1200 and 1250 °C.

TEM micrographs of precipitates are shown in Fig. 8. Fig. 8(a) shows a barium aluminotitanate precipitate. The selected area diffraction pattern for this precipitate is shown in Fig. 8(c). This precipitate has a composition close to BaAl<sub>2</sub>Ti<sub>5</sub>O<sub>14</sub> as measured by energy dispersive spectroscopy (EDS). However, due to the overlap of the Ba Lα and Ti Kα peaks, it is not possible to accurately measure the composition of barium titanate phases using EDS. Given the position of the compositions on the sub-solidus compatibility diagram of the BaO-Al<sub>2</sub>O<sub>3</sub>-TiO<sub>2</sub> system<sup>29</sup> and the previous WDS results this phase is likely to be Ba<sub>4</sub>Al<sub>2</sub>Ti<sub>10</sub>O<sub>27</sub>. Fig. 8(b) shows a barium aluminate precipitate of composition (as measured by EDS) BaAl<sub>2</sub>O<sub>4</sub>. The selected area diffraction pattern for this precipitate is shown in Fig. 8(d). The Ba<sub>4</sub>Al<sub>2</sub>Ti<sub>10</sub>O<sub>27</sub> precipitate has a rod morphology. The BaAl<sub>2</sub>O<sub>4</sub> precipitate has an irregular

morphology. Both precipitates have smooth (atomically rough) grain boundaries with the surrounding matrix grains.

The undoped series A sample has a mean matrix grain size of 1.3 μm as measured by SEM. The undoped series T sample has a mean matrix grain size of 0.8 μm. Additions of Al<sub>2</sub>O<sub>3</sub> cause the matrix grain size to decrease in both series of samples, with the grain size levelling out at 0.5–0.7 μm with addition of 0.5 mol% Al<sub>2</sub>O<sub>3</sub>. The matrix grain size is not greatly affected by the sintering temperature. The reduction in matrix grain size when Al<sub>2</sub>O<sub>3</sub> or TiO<sub>2</sub> is added may be due to solute segregation at the grain boundaries. Acceptor dopants are known to segregate at grain boundaries in BaTiO<sub>3</sub>.<sup>31</sup> Sharma et al.<sup>32</sup> found the solubility of TiO<sub>2</sub> in BaTiO<sub>3</sub> to be <0.1 mol%. Lee et al.<sup>33</sup> have shown that TiO<sub>2</sub> segregates to the grain boundaries in 0.2 mol% excess TiO<sub>2</sub>-BaTiO<sub>3</sub> during sintering at 1250 °C. Segregation of dopants at the grain boundaries may impede grain growth by the solute drag mechanism.<sup>34,35</sup>

#### 4. Discussion

The presence of abnormal grains and faceted matrix grains (Figs. 1–3 and 5) suggests that grain growth in the samples has taken place by an interface reaction-controlled mechanism.<sup>6,36–40</sup> In this mechanism, grain growth rate is negligible below a critical driving force and increases rapidly above the critical driving force. Therefore, only matrix grains

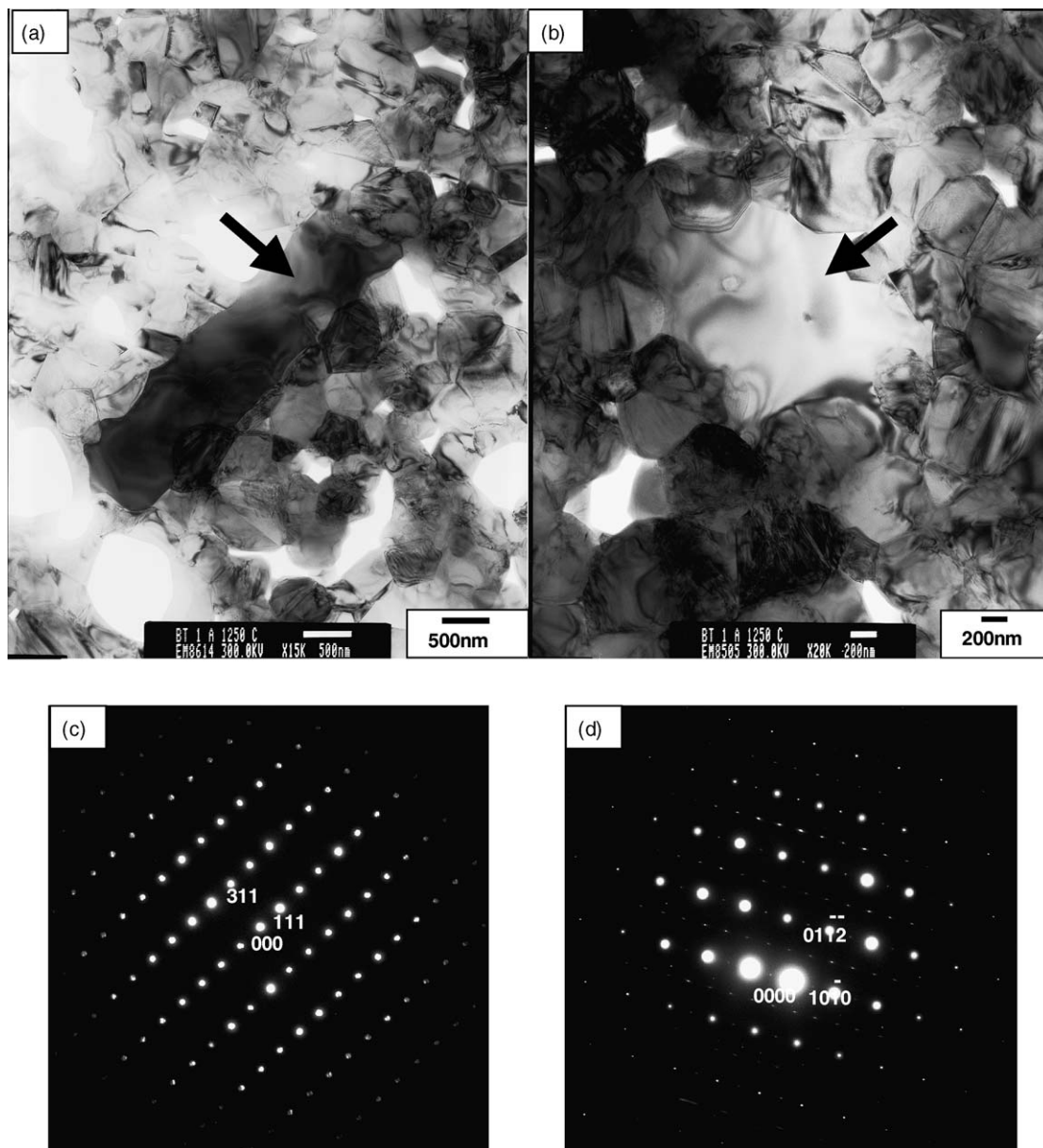


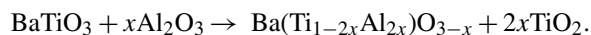
Fig. 8. TEM micrographs of: (a)  $\text{Ba}_4\text{Al}_2\text{Ti}_{10}\text{O}_{27}$  and (b)  $\text{BaAl}_2\text{O}_4$  precipitates in a 1.0 mol%  $\text{Al}_2\text{O}_3$ -doped sample sintered at  $1250^\circ\text{C}$  for 100 h. (c) and (d) are SADPs of the  $\text{Ba}_4\text{Al}_2\text{Ti}_{10}\text{O}_{27}$  and  $\text{BaAl}_2\text{O}_4$  precipitates, respectively.

above a certain critical size may grow, but they may do so very rapidly. In the particular case of  $\text{BaTiO}_3$ ,  $\{111\}$  twins provide low-energy sites for 2D nucleation which are necessary for AGG in samples sintered at temperatures  $\leq 1250^\circ\text{C}$ .<sup>5,6,14,15</sup> This leads to a bimodal structure of very fine matrix grains and coarse abnormal grains containing  $\{111\}$  twins.

The effect of  $\text{Al}_2\text{O}_3$  on abnormal grain growth in  $\text{BaTiO}_3$  can be explained by the presence of the second phases visible in the SEM and TEM micrographs and by considering the subsolidus compatibility diagram of the  $\text{BaO}-\text{Al}_2\text{O}_3-\text{TiO}_2$  system.<sup>29</sup> The compositions sintered in this work lie on the border of the  $\text{BaTiO}_3-\text{Ba}_6\text{Ti}_{17}\text{O}_{40}-\text{Ba}_4\text{Al}_2\text{Ti}_{10}\text{O}_{27}$  and  $\text{BaTiO}_3-\text{Ba}_4\text{Al}_2\text{Ti}_{10}\text{O}_{27}-\text{BaAl}_2\text{O}_4$  compatibility triangles.

The  $\text{BaTiO}_3$  powder used in this work had a slight excess of Ti (Table 1). This excess Ti reacts with  $\text{BaTiO}_3$  during sintering to form  $\text{Ba}_6\text{Ti}_{17}\text{O}_{40}$ ,<sup>1</sup> which acts as a site for the nucleation of  $\{111\}$  double twins.<sup>13,16</sup> These twins in turn provide non-vanishing sites for abnormal grain growth.<sup>5,14,15</sup> This is why the series A samples with 0 mol%  $\text{Al}_2\text{O}_3$  exhibit AGG.

When  $\text{Al}_2\text{O}_3$  is added to  $\text{BaTiO}_3$  it can enter into solid solution according to the reaction:



When 0.1 mol%  $\text{Al}_2\text{O}_3$  is added it appears that it is all able to enter into solid solution with  $\text{BaTiO}_3$ , as shown by the absence of  $\text{Al}_2\text{O}_3$ -containing second phases in the SEM



micrographs. This releases  $\text{TiO}_2$ , which can then react with  $\text{BaTiO}_3$  to form  $\text{Ba}_6\text{Ti}_{17}\text{O}_{40}$ . This causes an increase in the number of abnormal grains. When  $>0.1$  mol%  $\text{Al}_2\text{O}_3$  is added, it does not completely enter into solid solution. The free  $\text{Al}_2\text{O}_3$  can then react with  $\text{BaTiO}_3$ ,  $\text{Ba}_6\text{Ti}_{17}\text{O}_{40}$  and the  $\text{TiO}_2$  released from  $\text{BaTiO}_3$  by solid solution of  $\text{Al}_2\text{O}_3$ . The sample composition moves into the  $\text{BaTiO}_3$ – $\text{Ba}_4\text{Al}_2\text{Ti}_{10}\text{O}_{27}$ – $\text{BaAl}_2\text{O}_4$  compatibility triangle and the amount of  $\text{Ba}_6\text{Ti}_{17}\text{O}_{40}$  is reduced.  $\text{Ba}_4\text{Al}_2\text{Ti}_{10}\text{O}_{27}$  and  $\text{BaAl}_2\text{O}_4$  precipitates have atomically rough grain boundaries (Fig. 8) and so do not act as sites for formation of  $\{111\}$  twins.<sup>15,16</sup> Therefore, they cannot act as sites for abnormal grain growth. The removal of  $\text{Ba}_6\text{Ti}_{17}\text{O}_{40}$  causes the decrease in the number of abnormal grains. The  $\text{Al}_2\text{O}_3$  effectively scavenges excess  $\text{TiO}_2$  from the grain boundaries. Similar behaviour has been observed in  $\text{Na}_2\text{O}$ -<sup>21</sup> and  $\text{La}_2\text{O}_3$ -doped  $\text{BaTiO}_3$ <sup>11</sup> through the formation of  $\text{Na}_4\text{TiO}_4$  and  $\text{La}_2\text{Ti}_2\text{O}_7$  second phases, respectively.

In the T series samples with no added  $\text{Al}_2\text{O}_3$ , the number of abnormal grains is much greater than in the corresponding A series samples. Such behaviour has previously been observed in  $\text{BaTiO}_3$ <sup>41</sup> and is due to the 0.4 mol% excess  $\text{TiO}_2$ . This excess  $\text{TiO}_2$  reacts with  $\text{BaTiO}_3$  during sintering to form  $\text{Ba}_6\text{Ti}_{17}\text{O}_{40}$ , which promotes AGG as described above. During sintering of the T series samples with added  $\text{Al}_2\text{O}_3$ , rather than enter into solid solution in  $\text{BaTiO}_3$ ,  $\text{Al}_2\text{O}_3$  is able to react with the excess  $\text{TiO}_2$  and with  $\text{BaTiO}_3$  to form  $\text{Ba}_4\text{Al}_2\text{Ti}_{10}\text{O}_{27}$  and  $\text{BaAl}_2\text{O}_4$  precipitates. This prevents the excess  $\text{TiO}_2$  from forming  $\text{Ba}_6\text{Ti}_{17}\text{O}_{40}$  and accounts for the immediate drop in AGG when 0.1 mol%  $\text{Al}_2\text{O}_3$  is added. The change in shape of the abnormal grains from elongated to polyhedral in the samples sintered at 1250 °C also indicates that the excess  $\text{TiO}_2$  is being scavenged. Cho et al.<sup>17</sup> found that the grain shape of abnormal grains in  $\text{BaTiO}_3$  samples sintered at 1250 °C changed from elongated to polyhedral as the excess Ti fraction decreased from 0.5 to 0.3 at.%. This was due to a change in the stable grain boundaries of the abnormal grains from  $\{111\}$  boundaries to  $\{100\}$  boundaries.

The AGG behaviour of two of the compositions sintered at 1200 °C is unusual. The A series sample doped with 0.1 mol%  $\text{Al}_2\text{O}_3$  and the T series sample without  $\text{Al}_2\text{O}_3$  doping both have a higher abnormal grain number density when sintered at 1200 °C than when sintered at 1250 °C (Fig. 4). One would expect this behaviour to be the other way around because the critical driving force for grain growth should increase with sintering temperature decrease.<sup>37,39</sup> There are two possible reasons for this behaviour. The first is that the solubility of Ti in  $\text{BaTiO}_3$  has increased with sintering temperature. This will remove excess Ti and  $\text{Ba}_6\text{Ti}_{17}\text{O}_{40}$  from the grain boundaries and reduce AGG. The second possibility is that grain coalescence is taking place in the samples sintered at 1250 °C with the larger abnormal grains consuming the smaller abnormal grains. The abnormal grains in the samples sintered at 1250 °C are much larger than those in the samples sintered at 1200 °C, indicating that coalescence may have occurred. On the other hand, measurements of abnormal

grain number density of samples of these two compositions sintered at 1250 °C for 10 h show the number density to be similar to that of the samples sintered for 100 h. This indicates that coalescence has either not taken place or has taken place at sintering times  $<10$  h. Therefore, we cannot be certain of the reason for this unusual AGG behaviour at this time.

From the results it can be seen that addition of  $\text{Al}_2\text{O}_3$  can have a significant effect on abnormal grain growth in  $\text{BaTiO}_3$  samples sintered at temperatures  $\leq 1250$  °C. The effect that  $\text{Al}_2\text{O}_3$  has on AGG is dependant on the amount of excess  $\text{TiO}_2$  present in the samples. When no excess  $\text{TiO}_2$  is present, addition of  $\text{Al}_2\text{O}_3$  can cause an increase or a decrease in the amount of abnormal grain growth, depending on the amount of  $\text{Al}_2\text{O}_3$  added and the types of second phase formed. When  $\text{TiO}_2$  is added deliberately, addition of  $\text{Al}_2\text{O}_3$  effectively suppresses abnormal grain growth as a result of the formation of second phases, which do not act as nucleation sites for  $\{111\}$  twins.

## 5. Conclusions

$\text{BaTiO}_3$  has been doped with up to 1 mol%  $\text{Al}_2\text{O}_3$  and sintered at temperatures  $\leq 1250$  °C. Samples with and without additions of 0.4 mol%  $\text{TiO}_2$  were sintered. In the samples doped with  $\text{Al}_2\text{O}_3$  alone, addition of up to 0.1 mol%  $\text{Al}_2\text{O}_3$  promoted abnormal grain growth, while further additions of  $\text{Al}_2\text{O}_3$  inhibited it. Promotion of abnormal grain growth is caused by  $\text{Al}_2\text{O}_3$  dissolving in the  $\text{BaTiO}_3$  lattice and releasing  $\text{TiO}_2$ . This excess  $\text{TiO}_2$  reacts with  $\text{BaTiO}_3$  to form  $\text{Ba}_6\text{Ti}_{17}\text{O}_{40}$ , which acts as a nucleation site for  $\{111\}$  twins. These twins in turn act as low energy growth sites for abnormal grains. On further addition of  $\text{Al}_2\text{O}_3$ , inhibition of abnormal grain growth occurs by  $\text{Al}_2\text{O}_3$  reacting with  $\text{BaTiO}_3$ ,  $\text{Ba}_6\text{Ti}_{17}\text{O}_{40}$  and excess  $\text{TiO}_2$  to form  $\text{Ba}_4\text{Al}_2\text{Ti}_{10}\text{O}_{27}$  and  $\text{BaAl}_2\text{O}_4$ . Neither of these second phases have faceted grain boundaries, and so cannot form nucleation sites for  $\{111\}$  twins. Hence they do not act as growth sites for abnormal grains. In the samples doped with  $\text{Al}_2\text{O}_3$  and  $\text{TiO}_2$ , addition of  $\text{Al}_2\text{O}_3$  immediately causes a decrease in abnormal grain growth. This is due to the  $\text{Al}_2\text{O}_3$  reacting with the excess  $\text{TiO}_2$  and  $\text{BaTiO}_3$  to form  $\text{Ba}_4\text{Al}_2\text{Ti}_{10}\text{O}_{27}$  and  $\text{BaAl}_2\text{O}_4$ , thus preventing the formation of  $\text{Ba}_6\text{Ti}_{17}\text{O}_{40}$  by the excess  $\text{TiO}_2$ .

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